Foundations of Programming & Computing Lab

- **Wonyeol Lee.**

• POSTECH:	Assistant Professor in CS	(2024–Present)
• CMU:	Postdoc in CS	(2023–2024)
• Stanford:	PhD in CS	(2014–2017, 2020–2023)
KAIST:	Researcher in CS	(2017–2020)
• POSTECH:	BS in CS and Math	(2010–2014)

- **Research.**
  - PL: POPL (2023, 2020, 2018, 2014), PLDI (2025a, 2025b, 2016), CAV (2025).
  - ML: NeurIPS (2020-Spotlight, 2018), ICML (2025, 2023), ICLR (2024-Spotlight), AAAI (2020).

Research Interests

Mathematical Properties of **Programs and Computations**

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Mathematical Properties of **Programs and Computations**

Correctness **Efficiency** **Fundamental Limits** …

- Is a practically-used computation "correct" in any formal sense?
- Is there a more "efficient" computation that is correct?
- Is there any "fundamental limit" to achieving the computation?

Research Interests

Mathematical Properties of **Programs and Computations**

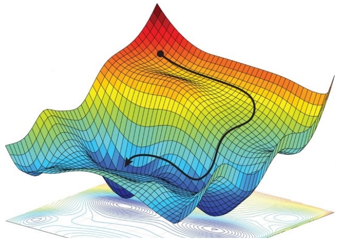
Continuous Values

$$6,\ 2.8,\ \frac{3}{7},\ \sqrt{5},\ \frac{\pi}{4},\ \ldots$$

Operations on Them

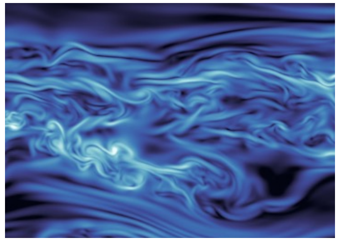
$$6 + 2.8,\ \frac{3}{7} \times \sqrt{5},\ \sin\left(\frac{\pi}{4}\right),\ \ldots$$

Machine Learning



Optimization

Computer Graphics

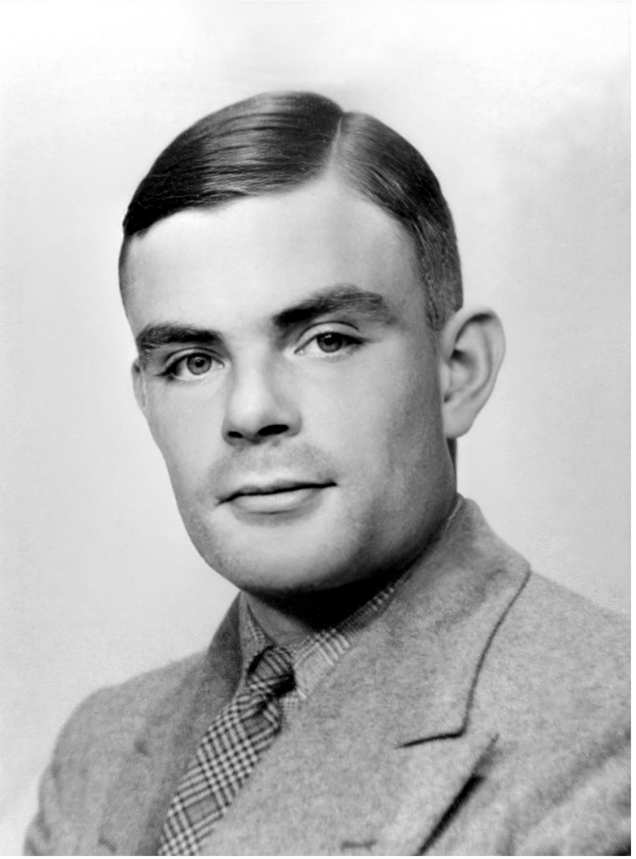


Scientific Computing

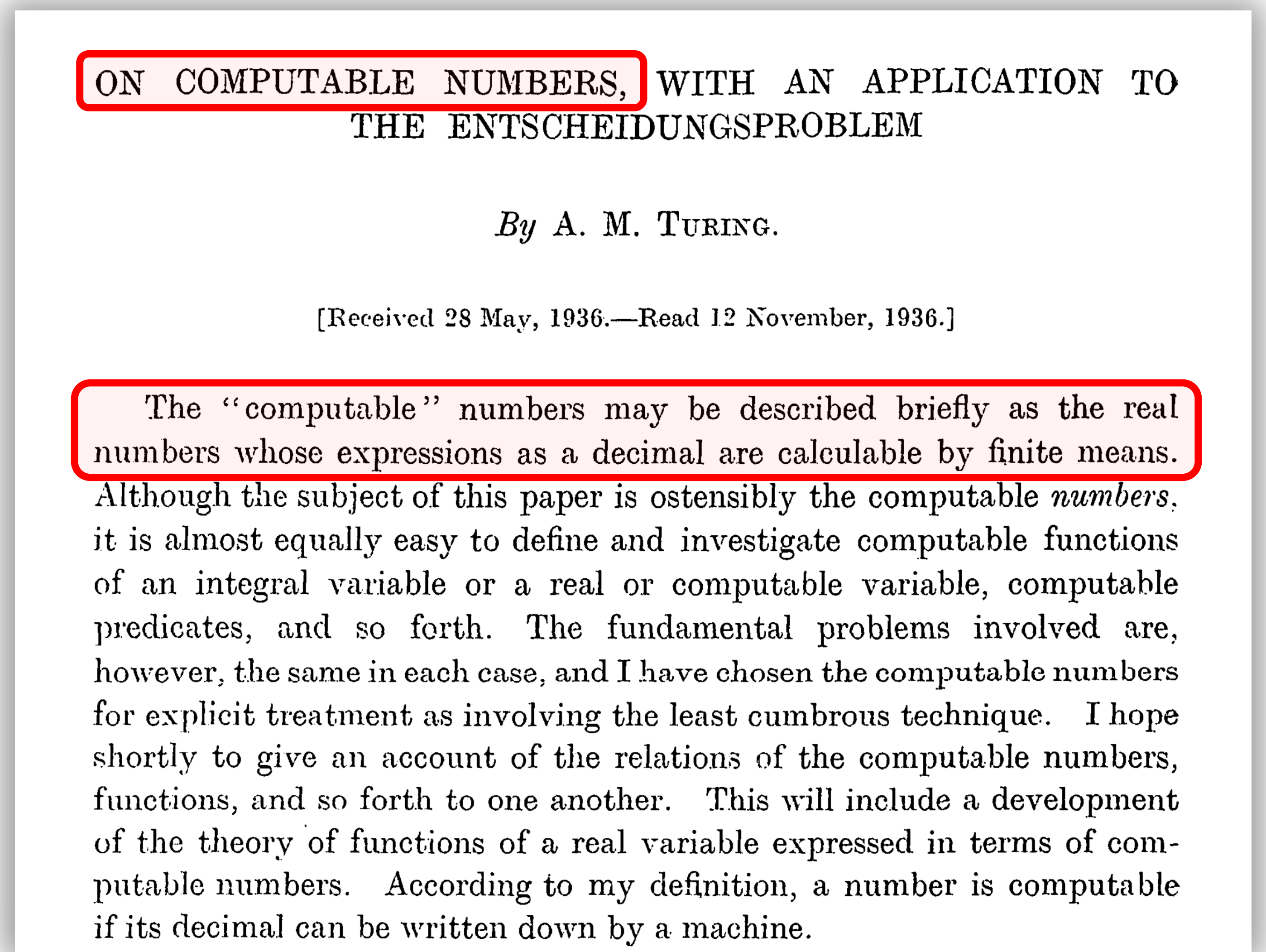
Differential Privacy

…

Early Days



Alan Turing



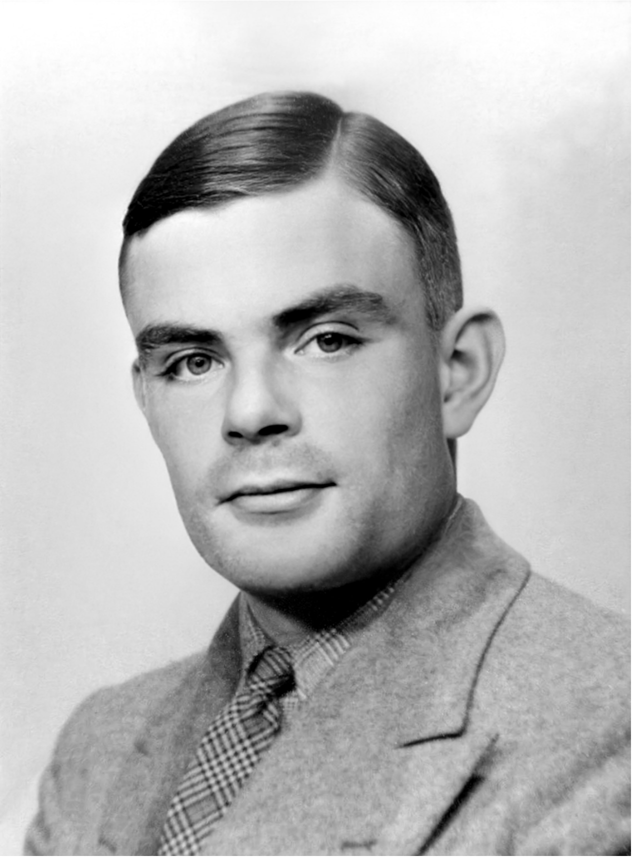
ON COMPUTABLE NUMBERS, WITH AN APPLICATION TO THE ENTSCHEIDUNGSPROBLEM

By A. M. Turing.

[Received 28 May, 1936.—Read 12 November, 1936.]

The "computable" numbers may be described briefly as the real numbers whose expressions as a decimal are calculable by finite means. Although the subject of this paper is ostensibly the computable *numbers*, it is almost equally easy to define and investigate computable functions of an integral variable or a real or computable variable, computable predicates, and so forth. The fundamental problems involved are, however, the same in each case, and I have chosen the computable numbers for explicit treatment as involving the least cumbrous technique. I hope shortly to give an account of the relations of the computable numbers, functions, and so forth to one another. This will include a development of the theory of functions of a real variable expressed in terms of computable numbers. According to my definition, a number is computable if its decimal can be written down by a machine.

Early Days



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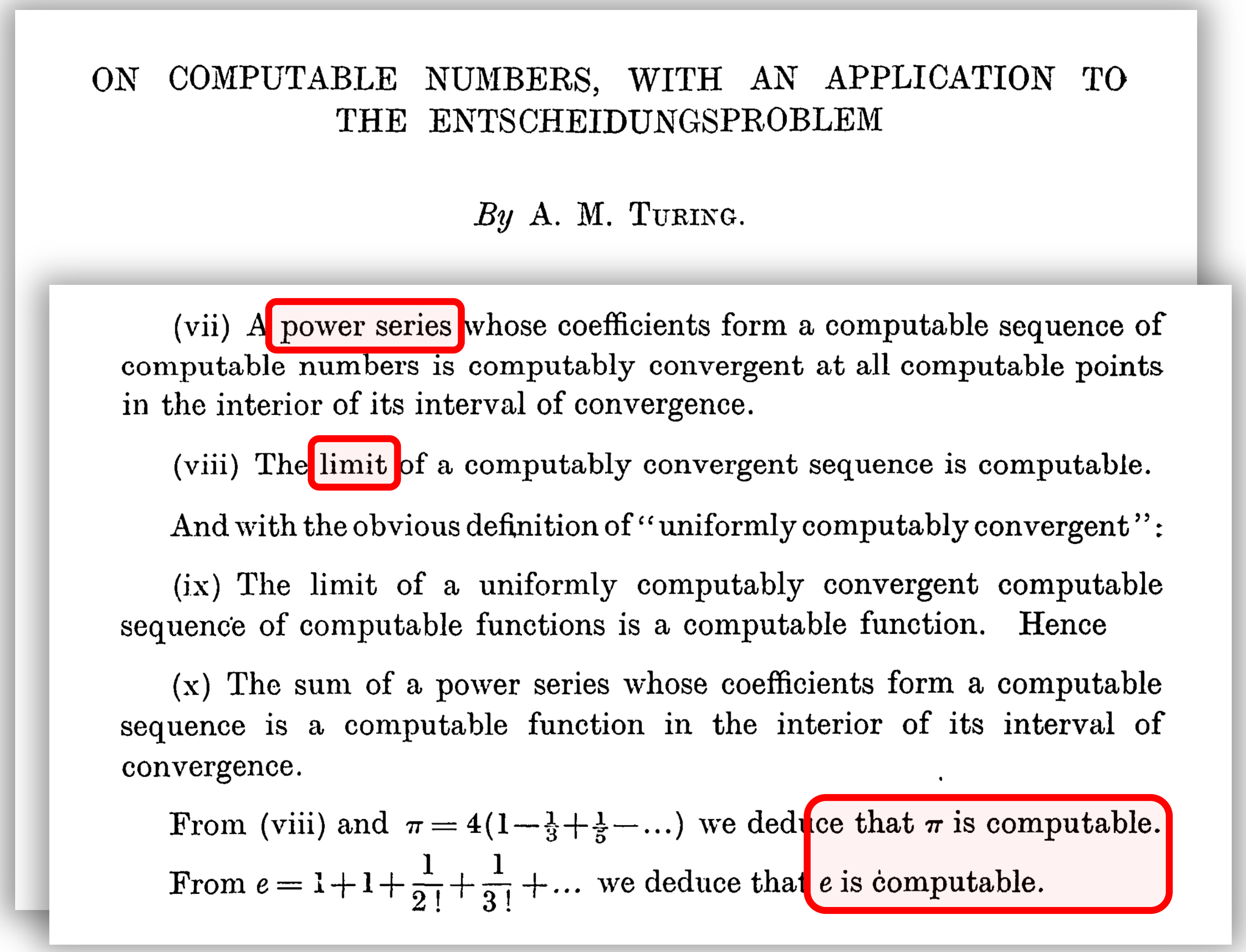

ON COMPUTABLE NUMBERS, WITH AN APPLICATION TO THE ENTSCHEIDUNGSPROBLEM

By A. M. TURING.

(vii) A power series whose coefficients form a computable sequence of computable numbers is computably convergent at all computable points in the interior of its interval of convergence.

(viii) The limit of a computably convergent sequence is computable.

And with the obvious definition of "uniformly computably convergent":

(ix) The limit of a uniformly computably convergent computable sequence of computable functions is a computable function. Hence

(x) The sum of a power series whose coefficients form a computable sequence is a computable function in the interior of its interval of convergence.

From (viii) and $\pi = 4(1-\frac{1}{3}+\frac{1}{5}-\dots)$ we deduce that π is computable.

From $e = 1+1+\frac{1}{2\,!}+\frac{1}{3\,!}+\dots$ we deduce that e is computable.

These Days

As a result of ~90 years of substantial efforts,

GNU Math/Scientific Library Intel MKL NumPy SciPy

TensorFlow PyTorch JAX Pyro Stan …

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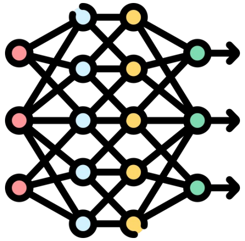
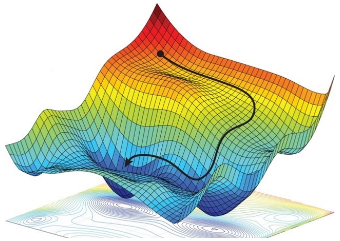


Machine Learning



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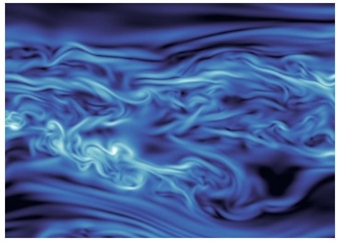

Scientific Computing

Differential Privacy

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Fundamental Computations

Function Evaluation	Compute $\sin(x)$.	GNU Math Library	Intel MKL	$\cdots$
Sample Generation	Sample from $\mathcal{N}(\mu, \sigma^2)$.	NumPy	SciPy	$\cdots$

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Integration **(≈ Probabilistic Inference)**	Compute $\int f(x)\, dx$.	Pyro	Stan	⋯
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Mathematically **Correct?**
Can Be More **Efficient?**
Any Fundamental **Limits?**

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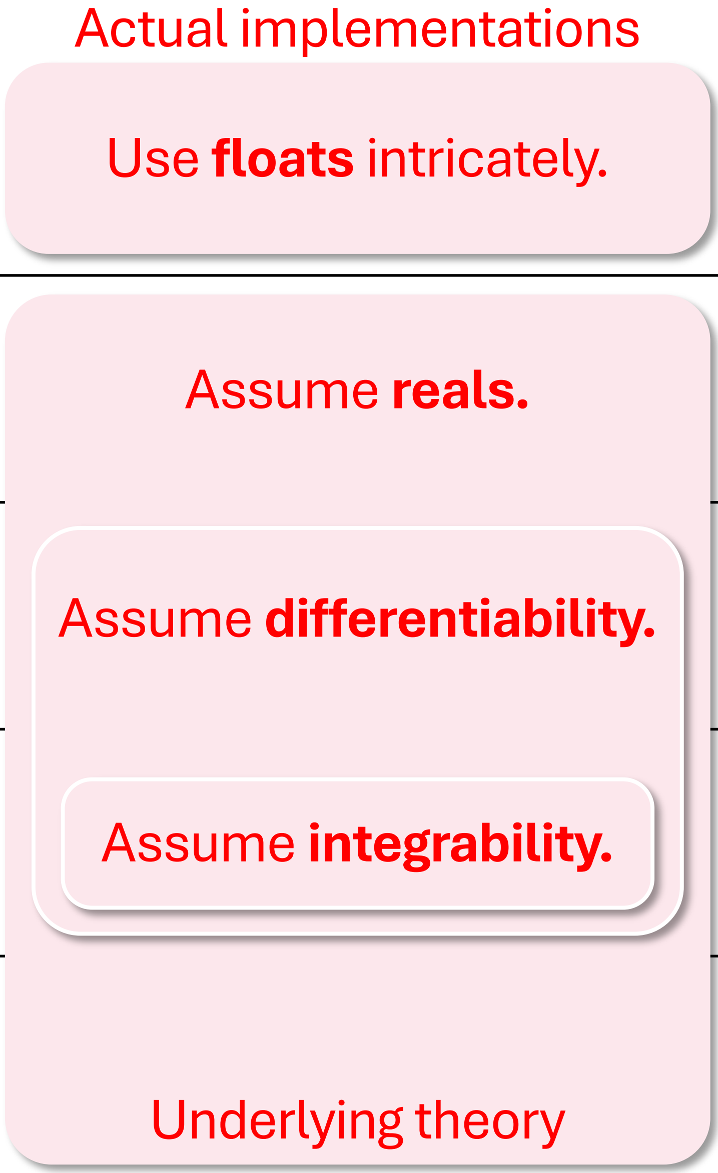

Actual implementations

Use **floats** intricately.

Assume **reals.**

Assume **differentiability.**

Assume **integrability.**

Underlying theory

Function Evaluation	GNU Math Library, intel Intel MKL, ···
Sample Generation	NumPy, SciPy, ···
Differentiation	TensorFlow, PyTorch, ···
Integration **(≈ Probabilistic Inference)**	Pyro, Stan, ···
Function Approximation	TensorFlow, PyTorch, ···

Our Works

(Dis)Prove **Correctness.**
Improve **Efficiency.**
Prove **Fundamental Limits.**

	Actual implementations	Programming Languages	Machine Learning
Function Evaluation	Use **floats** intricately.	[Ongoing 1] [Ongoing 2] [POPL 18] [PLDI 16]	
Sample Generation	Assume **reals.**	[Ongoing 1] [Ongoing 2] [PLDI 25a]	
Differentiation	Assume **differentiability.**		(Spotlight) [ICLR 24] [ICML 23] (Spotlight) [NeurIPS 20]
Integration **(≈ Probabilistic Inference)**	Assume **integrability.**	[Submitted] [PLDI 25b] [POPL 23] [POPL 20]	[AAAI 20] [NeurIPS 18]
Function Approximation	Underlying theory	[CAV 25]	[ICML 25]